

# CONSTANT LEVERAGE AND CONSTANT COST OF CAPITAL: A COMMON KNOWLEDGE HALF-TRUTH

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## ABSTRACT

A typical approach for valuing finite cash flows is to assume that leverage is constant (usually as target leverage) and the cost of equity,  $K_e$  and the Weighted Average Cost of Capital, WACC are also assumed to be constant. For cash flows in perpetuity, and with the cost of debt,  $K_d$  as the discount rate for the tax shield, it is indeed the case that the  $K_e$  and WACC applied to the FCF are constant if the leverage is constant. However this does not hold true for finite cash flows.

In this document we show that for finite cash flows,  $K_e$  and hence WACC

depend on the discount rate that is used to value the tax shield, TS and as expected,  $K_e$  and WACC are not constant with  $K_d$  as the discount rate for the tax shield, even if the leverage is constant. We illustrate this situation with a simple example. We analyze five methods: DCF using APV, FCF and traditional and general formulation for WACC, present value of CFE plus debt and Capital Cash Flow, CCF.

## KEY WORDS

WACC, constant cost of capital, constant leverage, cash flows.

**Clasificación JEL:** D61, G31, H43

## RESUMEN

Un enfoque típico para valorar flujos de caja finitos es suponer que el endeudamiento es constante (generalmente como un endeudamiento objetivo o deseado) y que por tanto, el costo del patrimonio,  $K_e$  y el costo promedio ponderado de capital CPPC, también son constantes. Para los flujos de caja perpetuos, y con el costo de la deuda,  $K_d$  como la tasa de descuento para el ahorro en impuestos o escudo fiscal,  $K_e$  y el CPPC aplicado al flujo de caja libre FCL son constantes si el endeudamiento es constante. Sin embargo esto no es verdad para los flujos de caja finitos.

En este documento mostramos que para flujos de caja finitos,  $K_e$  y por lo tanto el CPPC dependen de la tasa de

descuento que se utiliza para valorar el ahorro en impuestos, AI y según lo esperado,  $K_e$  y el CPPC no son constantes con  $K_d$  como la tasa de descuento para el ahorro en impuestos, aunque el endeudamiento sea constante. Ilustramos esta situación con un ejemplo simple. Analizamos cinco métodos: el flujo de caja descontado, FCD, usando APV, el FCD y la formulación tradicional y general del CPPC, el valor presente del flujo de caja del accionista, FCA más deuda y el flujo de caja de capital, FCC.

## PALABRAS CLAVE

Costo promedio ponderado de capital, CPPC, costo de capital constante, endeudamiento constante, flujos de caja.

## INTRODUCTION

In this document we show that using the findings of Tham and Velez-Pareja (2002), for finite cash flows,  $K_e$  (cost of levered equity) and hence WACC (Weighted Average Cost of Capital), depend on the discount rate that is used to value the tax shield (TS), and as expected,  $K_e$  and WACC are not constant with  $K_d$  (cost of debt) as the discount rate for the tax shield, even if the leverage is constant. We illustrate this situation with a simple example. We analyze five methods: DCF Discounted Cash flows (the Free Cash Flow, FCF) using APV, FCF and traditional and general formulation for WACC, present value of Cash Flow to equity (CFE) plus debt and Capital Cash Flow (CCF).

A typical approach for project or firm valuation which could be found in practice (See for example World Bank (2002), Benninga (1997, 2006), Brealey and Myers (2000, 2003), Brealey, Myers and Allen (2006), Copeland, Koller and Murrin (1995, 2000) is to discount cash flows expected within the finite time horizon at constant cost of capital, (usually as a target leverage) assuming that target leverage is maintained throughout the life of the project, and thus its cost of levered equity  $K_e$  and the WACC are constant. Though it might be convenient to perform calculations under such assumption, it is not in fact always true that  $K_e$  and WACC are constant under the constant leverage financing policy. As could be seen from the findings and example of Inselbag and Kaufold (1997), and as a general expression for  $K_e$  and WACC derived by Tham and Velez-Pareja (2002), both the cost of levered equity and

the Weighted Average Cost of Capital depend on the value of the interest tax shield ( $V^{TS}$ ), and in the case of finite cash flows valuation, they could be changing from period to period if certain choice is made for the rate to discount expected tax shields.

The case of variable leverage has been studied elsewhere by Mian and Velez Pareja (2008), Velez-Pareja (2004, 2005), Velez-Pareja and Burbano (2006), Velez-Pareja and Tham (2001, 2004, 2006a, 2006b), and Tham and Velez Pareja (2002, 2004). In these cases, they find complete consistency between all methods and with different assumptions about the discount rate for the tax shields.

Practitioners frequently assume that the risk (and corresponding discount rate,  $\psi$ ) of the interest tax shield is the cost of debt,  $K_d$ . This is done explicitly when, for example, the APV method is applied, or implicitly, if popular formula  $K_e = K_u + (K_u - K_d) \times (1 - T) \times D/E$  ( $K_u$ , the cost of unlevered equity;  $T$ , corporate tax rate;  $D$  and  $E$  are market values of debt and equity, respectively) is used to estimate the cost of equity capital. As Taggart (1991) and Tham and Velez-Pareja (2002, 2004) prove, this formulation is valid only for a fixed (in perpetuity) dollar amount of debt, thus under constant leverage assumption it could be applied only to perpetual cash flows. However, this formula is used by Fernandez (2002), Shapiro (2005) and others even within finite planning horizon and when dollar amount of debt is changing from period to period. Another example of implicit  $\psi = K_d$  assumption is applying Hamada's formulation to unlever and relever betas. Initially developed

by Hamada (1972) for flat perpetuity and risk free debt, his formula is persistently used in conjunction with discounting at constant WACC under constant leverage assumption,<sup>1</sup> potentially producing significant valuation errors as can be seen from the comprehensive example analyzed by Mian and Velez-Pareja (2008). Velez-Pareja and Tham (2004, 2006a, 2006b) repeatedly show that, if assumptions and formulae are mismatched, inconsistencies arise when calculating value with different methods. So analysts should be very careful dealing with finite cash flows. To obtain correct and consistent valuation results one should specify assumption for the risk of the tax shield ( $\psi$ ) first, and from that assumption choose the proper formulation for  $K_e$  and WACC.

Different values one proposes for  $\psi$ , the risk or discount rate for the TS might be questioned based on the particular debt policy and underlying expectations for the cash flow profile. However, when selecting the assumption or approach for  $\psi$ , we have to be consistent in the use of the formulation for the cost of capital. If under the constant leverage financing policy the risk  $\psi$  of the interest tax shield is assumed<sup>2</sup> to be equal to  $K_d$ , then  $K_e$  and WACC could not be assumed constant. Put it another way,  $\psi = K_d$  and constant  $K_e$  and WACC are incompatible assumptions within the constant leverage set up.

To illustrate the scenario of non constant cost of capital with constant leverage<sup>3</sup> we present a simple example, and analyze five DCF methods:

1. Adjusted present value (APV);
2. Discounting FCF at WACC calculated from the traditional formulation;
3. Discounting FCF at WACC calculated from the general formulation;<sup>4</sup>
4. Cash flow to equity (CFE) discounted at the cost of levered equity plus the value of debt, and
5. Capital Cash Flow (CCF) discounted at the corresponding cost of capital.

The rest of the document is organized as follows: in Section Two, we present the generalized formulation for the cost of capital for the finite cash flow valuation, and in particular formulae under the assumption that the discount rate for the tax shield (TS) is  $K_d$ . In Section Three we show a simple example. In Section Four we conclude.

## **I. GENERAL FORMULATION FOR $K_e$ AND WACC**

Taggart (1991) presents a revision of the set of formulations for the cost of levered  $K_e$  and WACC for perpetuities and finite cash flows. He introduces the formulation with and

1 Here we can mention very different texts from practitioners and academics: Pratt, Reilly and Schweihs (2000), Abrams (2001), Damodaran (2002),

2 This assumption is by itself debatable

3 Which for example could be achieved through debt rebalancing at the end of every period to keep constant its percentage of the estimated project value)

4 See Tham and Velez-Pareja (2002, 2004).

without personal taxes and for different level of risk for discounting the TS, including the Miles and Ezzell (1980). However, Taggart does not include the case of  $\psi = K_d$  for finite cash flows. Inselbag and Kaufold (1997) include the formulation of  $K_e$  and WACC for the case of  $K_d$ , the cost of debt as the level of risk for the TS and finite cash flows, but neither Taggart (1991) nor Inselbag and Kaufold (1997) show the formulation for the cost of capital appropriate to discount capital cash flow (CCF) under  $\psi = K_d$  and finite cash flows scenario.

Tham and Velez-Pareja (2002) present a derivation of the general expression for  $K_e$ , the cost of levered equity for different levels of  $\psi$  corresponding to the risk of the tax shields, and resulting formulations for the general WACC, which should be applied to discount the Free Cash Flow (FCF) and Capital Cash Flow (CCF) both for finite time horizon valuation and for perpetuities.

The general formulation for  $K_e$  is,

$$K_{e_i} = K_{u_i} + (K_{u_i} - K_{d_i}) \frac{D_{i-1}}{E_{i-1}^L} - (K_{u_i} - \psi_i) \frac{V_{i-1}^{TS}}{E_{i-1}^L}$$

Where  $K_u$  is the unlevered cost of equity,  $\psi$  is the risk (discount rate) of the TS,  $D$  is market value of debt,  $E$  is market value of equity and  $V^{TS}$  is the market value of TS;  $i$  is the period of analysis.

The general formulation for  $WACC^{FCF}$  is,

$$WACC_i^{FCF} = K_{u_i} - \frac{TS_i}{V_{i-1}^L} - (K_{u_i} - \psi_i) \frac{V_{i-1}^{TS}}{V_{i-1}^L}$$

where  $TS$  is tax savings,  $V^L$  is the market value of the levered firm and the other variables were defined above.

Following the path of the classic WACC derivation, we can easily show that general expression for the classic WACC is

$$WACC_i^{FCF} = \frac{E_{i-1}}{V_{i-1}^L} K_e + \frac{D_{i-1}}{V_{i-1}^L} K_d - \frac{TS_i}{V_{i-1}^L}$$

and we obtain traditional formula

$$WACC_i^{FCF} = E\%_{i-1} \times K_e + D\%_{i-1} \times K_d \times (1 - T)$$

when  $TS_i = K_d \times D_{i-1} \times T$ .

The general formulation for the  $WACC^{CCF}$  is

$$WACC_i^{CCF} = K_{u_i} - (K_{u_i} - \psi_i) \frac{V_{i-1}^{TS}}{V_{i-1}^L}$$

The general formula for the value of TS is

$$V_{i-1}^{TS} = \sum_{j=i}^N \frac{TS_j}{(1 + \psi)^j}$$

When the risk of TS,  $\psi$ , is  $K_u$ , then  $K_e$  simplifies to

$$K_{e_i} = K_{u_i} + (K_{u_i} - K_{d_i}) \frac{D_{i-1}}{E_{i-1}}$$

This formulation is valid for finite cash flows or perpetuities.

The WACC for the FCF simplifies to

$$WACC_i^{FCF} = K_{u_i} - \frac{TS_i}{V_{i-1}^L}$$

When taxes are paid when accrued and there is enough EBIT to earn the TS, then  $WACC^{FCF}$  is

$$WACC_i^{FCF} = K_{u_i} - \frac{T \times D_{i-1} \times K_d}{V_{i-1}^L}$$

$$= K_{u_i} - T \times D\%_{i-1} \times K_d$$

And the WACC for the CCF simplifies to

$$WACC_i^{CCF} = K_{u_i}$$

The value of the TS is

$$V_{i-1}^{TS} = \sum_{i=1}^N \frac{TS_i}{(1 + Ku)^i}$$

When the risk of TS,  $\psi$ , is  $Kd$ , then  $Ke$  simplifies to

$$Ke_i = Ku_i + (Ku_i - Kd_i) \left( \frac{D_{i-1}}{E_{i-1}} - \frac{V_{i-1}^{TS}}{E_{i-1}} \right)$$

for finite cash flows and

$$Ke_i = Ku_i + (Ku_i - Kd_i)(1 - T) \frac{D_{i-1}}{E_{i-1}}$$

for perpetuities.

The WACC for the FCF simplifies to

$$WACC_i^{FCF} = Ku_i - \frac{TS_i}{V_{i-1}^L} - [(Ku_i - Kd_i) \frac{V_{i-1}^{TS}}{V_{i-1}^L}]$$

for finite cash flows.

And the WACC for the CCF simplifies to

$$WACC^{CCF} = Ku_i - (Ku_i - Kd_i) \frac{V_{i-1}^{TS}}{V_{i-1}^L}$$

The value of the TS is

$$V_{i-1}^{TS} = \sum_{i=1}^N \frac{TS_i}{(1 + Kd)^i}$$

Observe that in the case of  $\psi = Ku$ ,  $Ke$  does not depend on TS and does not depend on the value of TS. Instead, when  $\psi = Kd$ ,  $Ke$  depends on TS and the value of TS. On the other hand, when  $\psi = Ku$ , WACC depends on TS and it will be constant when taxes are paid when accrued and there is enough EBIT to earn the TS. Instead, when  $\psi = Kd$ , WACC depends on TS and the value of TS.

From these formulations we can conclude that for finite cash flows leverage and cost of capital are constant when:

1. There is enough EBIT to fully earn the TS.
2. Taxes are paid when accrued.
3. The risk of TS is  $Ku$ .
4. Tax rate  $T$ , is constant.
5. Interest rate on debt is equal to the (market) cost of debt,  $Kd$ .

With this set of formulations we can illustrate with a simple numerical example that  $Ke$  and WACC are not constant when leverage is maintained constant if one assumes  $Kd$  (or any other value different from  $Ku$ ) to be the appropriate discount rate for the expected interest tax shields. This can be seen in the formulation for  $Ke$  and WACC.

Full consistency in valuation results could be obtained for all five methods we mentioned in the Introduction if proper formulation for the cost of capital is applied to discount corresponding cash flow.

## 2. SIMPLE EXAMPLE

A typical approach for valuing finite cash flows is to assume that leverage is constant and hence,  $Ke$  and WACC are introduced as constant. In this document, we show that using the findings of Tham and Velez-Pareja (2002),  $Ke$ , and hence WACC, depends on the value of the tax shield, TS, and as expected,  $Ke$  and WACC are not constant when we assume the risk of TS as  $Kd$ . We illustrate this situation with a simple example. We analyze five methods:

1. APV;
2. DCF using FCF and traditional formulation for WACC;
3. DCF using FCF and general formulation for WACC;

4. Present value of CFE plus debt, and
5. Capital Cash Flow, CCF discounted at the WACC for CCF.

In the Appendix, the reader will find the complete information and the financial statements.

Assume a project (or the firm) with the following information:

1. Some input data.
2. Income Statement, Cash Budget and Balance Sheet.
3. Cash flows derived from the financial statements.

Assume that the input data is as in Table 1.

Sales start with \$10 in year 1. The fixed assets cost \$4. The expected ending balance for the initial debt is shown in Table 2.

The complete financial statements are shown in the Appendix. From the financial statements we derive the following cash flows:

1. Free Cash Flow, FCF.
2. Cash Flow to Debt, CFD.
3. Cash Flow to Equity, CFE.
4. Tax savings.

**Table 1.** Input Data

	Year 1	Year 2	Year 3	Year 4
Real increase in sales	2%	2%	2%	2%
Inflation rate	6%	6%	6%	6%
Tax rate	35%	35%	35%	35%
Cost of debt $K_d$	11.20%	11.20%	11.20%	11.20%
Unlevered return, $K_u$	15.10%	15.10%	15.10%	15.10%
Risk of TS, $K_d$	11.20%	11.20%	11.20%	11.20%
D%	60%	60%	60%	60%
E%	40%	40%	40%	40%
Accounts receivable AR	10%	10%	10%	10%
Accounts payable AP	5%	5%	5%	5%
Gross margin	40%	40%	40%	40%

**Table 2.** Final debt schedule

	Year 0	Year 1	Year 2	Year 3	Year 4
Ending debt balance	3.14	2.78	1.97	1.04	
Principal payment		0.36	0.81	0.93	1.04
Interest payment		0.35	0.31	0.22	0.12

Now we calculate the working capital for each year.

**Table 3a.** Working capital

Year	1	2	3	4
Accounts Receivable	1.00	1.08	1.17	1.26
Accounts Payable	0.30	0.32	0.35	0.38
Working Capital, WC	0.70	0.76	0.82	0.88
Change in WC	0.70	0.06	0.06	0.07

With the change in working capital we can construct the FCF using the indirect method.

**Table 3b.** Calculation of the FCF

Year	1	2	3	4
EBIT(1-T)	1.95	2.00	2.02	2.02
Depreciation	1.00	1.25	1.56	1.95
Change in WC	-0.70	-0.06	-0.06	-0.07
Purchase of assets	-1.00	-1.25	-1.56	-1.95
FCF	1.25	1.94	1.96	1.95

We show the different cash flows in the next table.

**Table 4.** Cash flows derived from the Financial Statements with Debt as % of Market Value

Year	1	2	3	4
FCF	1.25	1.94	1.96	1.95
CFD from the CB	0.72	0.72	0.71	0.67
TS from the CB	0.12	0.11	0.08	0.04
CFE from the CB	0.67	0.92	0.89	0.84
CFE = FCF + TS - CFD	0.67	0.92	0.89	0.84

With this information we can perform the valuation of the cash flows. In the next tables we show the valuation for each method after solving the itera-

tion process to solve the circularity between value and discount rate, where necessary.



Using the APV and assuming that the discount rate for the TS is  $K_d$ :

**Table 5.** Adjusted Present Value, APV

Year	0	1	2	3	4
FCF		1.25	1.94	1.96	1.95
TS		0.12	0.11	0.08	0.04
APV					
PV(FCF at $K_u$ )	4.95	4.45	3.18	1.69	
PV(TS at $K_d$ )	0.28	0.19	0.10	0.04	
APV	5.23	4.64	3.28	1.73	
Value of debt, D	3.14	2.78	1.97	1.04	
Leverage, D%	60%	60%	60%	60%	

Using the DCF, the traditional WACC, and assuming that the discount rate for the TS is  $K_d$ :

**Table 6.** Traditional WACC

Year	0	1	2	3	4
FCF		1.25	1.94	1.96	1.95
Weight of debt D%	60.00%	60.00%	60.00%	60.00%	
After tax cost of debt		7.28%	7.28%	7.28%	7.28%
Contribution of debt to WACC		4.37%	4.37%	4.37%	4.37%
Equity					
Relative weight of equity E%	40.00%	40.00%	40.00%	40.00%	
$K_e = K_u + (K_u - K_d)(D\%/E\% - VTS/E)$		20.43%	20.55%	20.65%	20.75%
Contribution of equity to WACC		8.17%	8.22%	8.26%	8.30%
After tax WACC		12.54%	12.59%	12.63%	12.67%
Value	5.23	4.64	3.28	1.73	
Value of debt, D	3.14	2.78	1.97	1.04	
Leverage, D%	60%	60%	60%	60%	

Observe that  $K_e$  and WACC are not constant. This occurs because the  $K_e$  is a function of the value of TS.

Using the DCF, the general WACC, and assuming that the discount rate for the TS is  $K_d$ :

**Table 7.** General WACC

Year	0	1	2	3	4
FCF		1.25	1.94	1.96	1.95
After tax WACC (general)					
$WACC_i^{FCF} = Ku_i - \frac{TS_i}{V_{i-1}^L} - [(Ku_i - Kd_i) \frac{V_{i-1}^{TS}}{V_{i-1}^L}]$		12.54%	12.59%	12.63%	12.67%
V	5.23	4.64	3.28	1.73	
Value of debt, D	3.14	2.78	1.97	1.04	
Leverage, D%	60%	60%	60%	60%	

Using the CFE and assuming that the discount rate for the TS is Kd:

**Table 8.** Value Calculation with CFE

Year	0	1	2	3	4
Ending debt balance	3.14	2.78	1.97	1.04	
Tax shield TS		0.09	0.07	0.05	0.02
PV(TS at Kd)	0.28	0.19	0.10	0.04	
CFE= FCF + TS – CFD (with market value of debt)		0.67	0.92	0.89	0.84
Relative weight of debt D%	60.0%	60.0%	60.0%	60.0%	
Relative weight of equity E%	40.0%	40.0%	40.0%	40.0%	
$Ke = Ku + (Ku-Kd)(D\%/E\% - VTS/E)$		20.43%	20.55%	20.65%	20.75%
Value of equity	2.09	1.85	1.31	0.69	
Value of debt	3.14	2.78	1.97	1.04	
Total value	5.23	4.64	3.28	1.73	-
Leverage, D%		60%	60%	60%	60%

And finally, using the CCF with the  $WACC^{CCF}$ , and assuming that the discount rate for the TS is Kd:

**Table 9.** Capital Cash Flow

Year	0	1	2	3	4
FCF		1.25	1.94	1.96	1.95
TS		0.12	0.11	0.08	0.04
CCF		1.37	2.05	2.04	1.99
WACC for CCF:					
$WACC^{CCF} = Ku_i - (Ku_i - Kd_i) \frac{V_i^{TS}}{V_i^L}$		14.89%	14.94%	14.98%	15.02%
Value	5.23	4.64	3.28	1.73	0.00
Value of debt, D	3.14	2.78	1.97	1.04	
Leverage, D%	60%	60%	60%	60%	

Observe that the  $WACC^{CCF}$  is not constant even if we assume that  $Ku$  is constant.  $WACC^{CCF}$  is constant and equal to  $Ku$  (assuming no change in the operating risk for the firm and constant inflation) when we assume that the discount rate for the TS is  $Ku$ . The reason is identical to the one that makes  $Ke$  and  $WACC^{FCF}$  non constant when leverage is constant and we assume the risk of TS equal to  $Kd$ :  $Ke$ ,  $WACC^{FCF}$  and  $WACC^{CCF}$  depend on the value of TS.

As we have shown, first of all, all methods match<sup>5</sup> when we use the proper formulation for the cost of capital ( $Ke$  and  $WACC$ ); second, we have shown that the constant leverage does not mean that  $Ke$  and  $WACC$  are constant. Tham and Velez-Pareja (2002, 2005), and Velez-Pareja and Tham (2006a, 2006b) have shown

that when using  $Ku$  as the risk for the TS and some conditions regarding the payment of taxes, the existence of enough EBIT to earn the TS and the source of the TS, the cost of capital is constant.<sup>6</sup>

Observe that the value calculated assuming  $\psi$  equal to  $Kd$  is higher than the value when we assume that  $\psi$  equal to  $Ku$ . A question arises here: is it reasonable to think that, changing the financing policy from constant leverage to predetermined debt schedule (non constant leverage), the firm will increase its value? We leave the answer to this question for another work.

Now we can check the difference between the initial debt schedule and the new debt schedule based on the market value of debt.

5 This is a matching of identical results. We have tested it for more than 10 decimals and the difference is strictly, zero. The interested reader might receive the spreadsheet upon request to the authors.

6 We are assuming that  $EBIT \geq 0$ . When this condition is not met, the traditional expression for  $WACC$  is no longer valid. When this happens we should use equation (4) and as can be seen from it,  $WACC$  might not be constant (imagine that there is not TS earned during some period). When TS is not earned,  $WACC$  for the FCF is just  $Ku$ .

**Table 10.** Initial debt schedule

Year	0	1	2	3	4
Ending debt balance	3.75	2.45	1.50	0.75	
Principal payment		1.30	0.95	0.75	0.75
Interest payment		0.42	0.27	0.17	0.08

The new debt schedule is shown in next table.

**Table 11.** Debt Based on  $D\% \times \text{Market Value of firm}$ 

Year	0	1	2	3	4
Ending debt balance	3,14	2,78	1,97	1,04	
Principal payment		0,36	0,81	0,93	1,04
Interest payment		0,35	0,31	0,22	0,12

This means that management has to adjust debt from the beginning in order to achieve the target leverage. The difference in debt level is as follows.

**Table 12.** Difference of Market Value Debt with Initial Debt

Year	0	1	2	3	4
Difference with initial debt	0.61	-0.33	-0.47	-0.29	0.00

### 3. SUMMARY AND CONCLUDING REMARKS

We have shown that a constant leverage does not grant that the cost of capital is constant when the risk of the TS is  $K_d$ . Moreover, in order to achieve a proper valuation of finite cash flows with a constant leverage when the risk of TS is  $K_d$ , we have to use some formulations that differ from the traditional used by practitioners and textbooks. In other words, assuming constant leverage is not a sufficient condition to have constant cost of capital. We need to make explicit assumptions on the risk for the TS and use formulation for the cost of capital that is consistent with the assumed risk of the tax shield. Using the proper formulation in this scenario, we obtain full consistency in the calculation of value. This means that there are no advantages of one

method over another. All of them give the same value (when properly done) and all of them (even the APV) require iterations when the risk of the TS is  $K_d$ .

In short, we can conclude that for finite cash flows leverage and cost of capital are constant when:

1. There is enough EBIT to fully earn the TS.
2. Taxes are paid when accrued.
3. The risk of TS is  $K_u$ .
4. Tax rate  $T$ , is constant.
5. Interest rate on debt is equal to the (market) cost of debt,  $K_d$ .

In addition, we have to be aware that performing cash flow valuation with constant  $K_e$  and WACC under constant leverage assumption implies that particular formulations must be

used for the estimation of  $K_e$ . Since the possibility of constant leverage and constant cost of capital scenario arises only when  $\psi = K_u$ , analysts should use formula

$$K_{e_i} = K_{u_i} + (K_{u_i} - K_{d_i}) \frac{D_{i-1}}{E_{i-1}^L}$$

to calculate the cost of levered equity directly, and formula

$$\beta_{u_i} = \frac{\beta_{Lev_i} + \beta_{d_i} \frac{D_{i-1}}{E_{i-1}^L}}{1 + \frac{D_{i-1}}{E_{i-1}^L}}$$

or

$$\beta_{Lev_i} = \beta_{u_i} + (\beta_{u_i} - \beta_{d_i}) \frac{D_{i-1}}{E_{i-1}^L}$$

for unlevering and levering the beta in case they use the CAPM. Here  $\beta_u$  and  $\beta_{Lev}$  are the unlevered and levered  $\beta$ 's and  $D_{t-1}$  and  $E_{t-1}$  are the market values of debt and equity.

When we assume that the risk of the TS is  $K_d$ , we cannot assume constant WACC or  $K_e$  because leverage is constant. The formulations for  $K_e$  and WACC (either for the Free Cash Flow, FCF or the Capital Cash Flow, CCF) depend not only on the constant leverage, but on the value of the TS and that refrains the cost of capital from being constant even if leverage is constant.

Summary of results assuming  $K_d$  as the risk of the TS, Table 13.

In the case of the risk for the TS equal to  $K_u$ , we can observe the equations for  $K_e$ , WACC for the FCF and the CCF, as follows:

The general formulation for  $K_e$  is,

$$K_{e_i} = K_{u_i} + (K_{u_i} - K_{d_i}) \frac{D_{i-1}}{E_{i-1}^L} - (K_{u_i} - \psi_i) \frac{V_{i-1}^{TS}}{E_{i-1}^L}$$

When the risk of the TS is  $K_u$ , the third term of the RHS of the equation

**Table 13.** WACC for different approaches assuming constant leverage (60%)

Year	1	2	3	4
1. APV = PV(FCF at $K_u$ ) + PV(TS at $K_d$ ) WACC	ND	ND	ND	ND
2. Traditional WACC After tax WACC $K_d \times D\% \times (1-T) + K_e \times E\%$	12.54%	12.59%	12.63%	12.67%
3. General WACC  $WACC_i^{FCF} = K_{u_i} - \frac{TS_i}{V_{i-1}^L} - [(K_{u_i} - K_{d_i}) \frac{V_{i-1}^{TS}}{V_{i-1}^L}]$	12.54%	12.59%	12.63%	12.67%
4. $V = PV(CFE \text{ at } K_e) + D$ $K_e = K_u + (K_u - K_d)(D\%/E\% - V^{TS}/E)$	20.43%	20.55%	20.65%	20.75%
5. Capital Cash Flow, CCF:  $WACC^{CCF} = K_{u_i} - (K_{u_i} - K_{d_i}) \frac{V_{i-1}^{TS}}{V_{i-1}^L}$	14.89%	14.94%	14.98%	15.02%

vanishes and  $K_e$  depends only on  $K_u$ ,  $K_d$  and leverage (constants). Hence,  $K_e$  is constant.

The general formulation for  $WACC^{FCF}$  is,

$$WACC_i^{FCF} = K_{u_i} - \frac{TS_i}{V_{i-1}^L} - (K_{u_i} - \psi_i) \frac{V_{i-1}^{TS}}{V_{i-1}^L}$$

When the risk of the TS is  $K_u$ , the third term in the previous equation vanishes and the second term is  $T \times K_d \times D\%$  and hence  $WACC^{FCF}$  depends only on leverage which is constant. Hence, WACC is constant.

$$WACC_i^{FCF} = \frac{E_{i-1}}{V_{i-1}^L} K_e + \frac{D_{i-1}}{V_{i-1}^L} K_d - \frac{TS_i}{V_{i-1}^L}$$

In this case, if  $K_e$  is constant then  $WACC^{FCF}$  is constant.

The general formulation for the  $WACC^{CCF}$  is

$$WACC_i^{CCF} = K_{u_i} - [(K_{u_i} - \psi_i) \frac{V_{i-1}^{TS}}{V_{i-1}^L}]$$

When the risk of the TS is  $K_u$ , the second term of the RHS of the equation vanishes and  $WACC^{CCF} = K_u$  which is a constant.

## Appendix A

### Financial Statements and Cash Flows

**Table A1.** Input Data

Real increase in sales	2%			
Inflation rate	6%	D%	60%	
Tax rate	35%	E%	40%	
Cost of debt $K_d$	11.20%	$K_e$	20.95%	
$K_u$ = Unlevered return	15.10%			
<b>Year</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
Unlevered return	15.10%	15.10%	15.10%	15.10%
$\psi = K_d$	11.20%	11.20%	11.20%	11.20%
Inflation	6%	6%	6%	6%
Real increase in sales	2%	2%	2%	2%
Accounts Receivable	10%	10%	10%	10%
Accounts Payable	5%	5%	5%	5%
Gross margin	40%	40%	40%	40%

**Table A2a.** Initial Income Statement

<b>Year</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
Sales revenues	10.00	10.81	11.69	12.64
COGS	-6.00	-6.49	-7.01	-7.58
Gross income	4.00	4.32	4.68	5.06
Depreciation	-1.00	-1.25	-1.56	-1.95
EBIT	3.00	3.07	3.11	3.10
Interest payments	-0.42	-0.27	-0.17	-0.08
EBT	2.58	2.80	2.95	3.02
Taxes	-0.90	-0.98	-1.03	-1.06
Net income	1.68	1.82	1.91	1.96

**Table A2b.** Final Income Statement (Debt as % of Market Value)

<b>Year</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
Sales revenues	10.00	10.81	11.69	12.64
COGS	-6.00	-6.49	-7.01	-7.58
Gross income	4.00	4.32	4.68	5.06
Depreciation	-1.00	-1.25	-1.56	-1.95
EBIT	3.00	3.07	3.11	3.10
Interest payments	-0.35	-0.31	-0.22	-0.12
EBT	2.65	2.76	2.89	2.99
Taxes	-0.93	-0.97	-1.01	-1.05
Net income	1.72	1.80	1.88	1.94

**Table A3a.** Initial Balance Sheet

<b>Year</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
Accounts Receivable AR		1.00	1.08	1.17	1.26
Fixed assets	4.00	5.00	6.25	7.81	9.77
Cumulated depreciation		-1.00	-2.25	-3.81	-5.77
Net fixed assets	4.00	4.00	4.00	4.00	4.00
Total assets	4.00	5.00	5.08	5.17	5.26
Accounts Payable AP		0.30	0.32	0.35	0.38
Debt	3.75	2.45	1.50	0.75	
Equity	0.25	0.25	0.25	0.25	0.25
New equity		2.00	1.01	0.81	0.82
Repurchase of equity					
Net equity	0.25	2.25	3.26	4.07	4.88
Total liabilities and equity	4.00	5.00	5.08	5.17	5.26



**Table A3b.** Final Balance Sheet (Debt as % of Market Value)

Year	0	1	2	3	4
Accounts Receivable AR		1.00	1.08	1.17	1.26
Fixed assets	4.00	5.00	6.25	7.81	9.77
Cumulated depreciation		-1.00	-2.25	-3.81	-5.77
Net fixed assets	4.00	4.00	4.00	4.00	4.00
Total assets	4.00	5.00	5.08	5.17	5.26
Accounts Payable AP		0.30	0.32	0.35	0.38
Debt	3.14	2.78	1.97	1.04	
Equity	0.86	0.86	0.86	0.86	0.86
New equity	0.00	1.06	1.93	2.92	4.02
Repurchase of equity					
Net equity	0.86	1.92	2.79	3.78	4.88
Total liabilities and equity	4.00	5.00	5.08	5.17	5.26

**Table A4a.** Initial Cash Budget

Year	0	1	2	3	4
Cash Budget					
Accounts Receivable		0.00	1.00	1.08	1.17
Sales revenues no credit		9.00	9.73	10.52	11.38
Suppliers payments		-5.70	-6.16	-6.66	-7.20
Accounts payable		0.00	-0.30	-0.32	-0.35
Purchase of assets	-4.00	-1.00	-1.25	-1.56	-1.95
Taxes		-0.90	-0.98	-1.03	-1.06
Net Cash Balance NCB	-4.00	1.40	2.04	2.02	1.98
Loan inflow	3.75				
Loan principal payment		-1.30	-0.95	-0.75	-0.75
Interest payment		-0.42	-0.27	-0.17	-0.08
NCB	3.75	-1.72	-1.22	-0.92	-0.83
Initial equity investment	0.25				
New (repurchase of) equity		2.00	1.01	0.81	0.82
Repurchase of equity					
Dividends		-1.68	-1.82	-1.91	-1.96
NCB	0.25	0.32	-0.81	-1.10	-1.15
Net cumulated balance	-	-	-	-	-

**Table A4b.** Final Cash Budget (Debt as a % of Market Value)

Year	0	1	2	3	4
Cash Budget		0.00	1.00	1.08	1.17
Accounts Receivable		9.00	9.73	10.52	11.38
Sales revenues no credit		-5.70	-6.16	-6.66	-7.20
Suppliers payments		0.00	-0.30	-0.32	-0.35
Accounts payable	-4.00	-1.00	-1.25	-1.56	-1.95
Purchase of assets		-0.93	-0.97	-1.01	-1.05
Taxes	-4.00	1.37	2.05	2.04	1.99
Net Cash Balance NCB	3.14				
Loan inflow		-0.36	-0.81	-0.93	-1.04
Loan principal payment		-0.35	-0.31	-0.22	-0.12
Interest payment	3.14	-0.71	-1.13	-1.15	-1.15
NCB	0.86				
Initial equity investment		1.06	0.87	0.99	1.11
New (repurchase of) equity					
Repurchase of equity		-1.72	-1.80	-1.88	-1.94
Dividends	0.86	-0.67	-0.92	-0.89	-0.84
NCB	0.00	0.00	0.00	0.00	0.00
Net cumulated balance		0.00	1.00	1.08	1.17

**Table A5a.** Working Capital

Year	0	1	2	3	4
AR	0.00	1.00	1.08	1.17	1.26
AP	0.00	0.30	0.32	0.35	0.38
WC	0.00	0.70	0.76	0.82	0.88
Change in WC		0.70	0.06	0.06	0.07

**Table A5b.** Cash Flow Calculation

Year	0	1	2	3	4
EBIT(1-T)		1.95	2.00	2.02	2.02
Depreciation		1.00	1.25	1.56	1.95
Change in WC		-0.70	-0.06	-0.06	-0.07
Purchase of assets		-1.00	-1.25	-1.56	-1.95
FCF		1.25	1.94	1.96	1.95

**Table A6a.** Initial debt schedule

<b>Year</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
Ending debt balance	3.75	2.45	1.50	0.75	
Principal payment		1.30	0.95	0.75	0.75
Interest payment		0.42	0.27	0.17	0.08

**Table A6b.** Debt schedule Based on Market Value

<b>Year</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
Ending debt balance market value	3.14	2.78	1.97	1.04	
Principal payment		0.36	0.81	0.93	1.04
Interest payment		0.35	0.31	0.22	0.12

**Table A7.** Cash Flow Calculations

<b>Year</b>	<b>0</b>	<b>1</b>	<b>2</b>	<b>3</b>	<b>4</b>
Tax shield		0.12	0.11	0.08	0.04
CFD		0.71	1.13	1.15	1.15
CFE from the CB = Dividends – New Equity		0.67	0.92	0.89	0.84
FCF = CFD+CFE-TS		1.25	1.94	1.96	1.95
CFE = FCF + TS - CFD		0.67	0.92	0.89	0.84

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